Preparing High School Students for Success in Trigonometry

EDCI 528-002

Solutions and Evaluation Plan

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Performance System

Each August instructors and students return to the halls of high schools everywhere to begin a new year of learning. A unique event for many juniors/seniors is a course called Precalculus which begins with a semester of…Trigonometry.

A significant challenge of Trigonometry is that the majority of the mathematical operations involved are based on basic skills briefly encountered in Geometry, a course which the students completed 15 months prior. During 9 of those 15 months the students were focused on Algebra II, which currently rarely references their Trigonometry-related Geometry knowledge. The remaining 6 months comprise two summer breaks of 3 months each during which many students receive little exposure to math.

Consequently, the students often enter Trigonometry attempting to reference background knowledge that many find difficult to access. Compounding this situation, Precalculus is an honors-level math course tying together a wide array of previously taught concepts with minimal class time available for readdressing critical background knowledge, which is expected to be in place when the students enter the classroom.

Thereby the conundrum is created. Does the instructor omit instruction and practice regarding two important Precalculus concepts at the end of the school year and spend the first two weeks reviewing and reteaching previous skills and knowledge? Or does the instructor drive ahead and provide background knowledge support tools for the students to pursue concurrent with their Precalculus work, outside of class?

Historically the latter option is typically selected. A strong 'college/career readiness' case is made for this direction as these students are in their last two years of high school and there will be many college and/or career experiences ahead where they are given new tools to learn to use to accomplish a task, and they will differentiate themselves by what problem-solving skills they bring to that task, and how well they can adapt. This is career training.

Drivers and Performance Gaps

Trigonometry is 'sold' to the students as a venue for learning problem-solving skills. They are told on Day 1 that 20% of them will use these math tools the following year in Calculus and then 15% will use Trigonometry beyond high school, but 100% of them will use the problem-solving skills built this year by learning how to be given new tools and then figuring out how to complete a task using those tools. The jobs of tomorrow require employees to take tools that may be new to them, and use those tools to address needs.

Students at many high schools are expected to successfully complete Precalculus before they graduate. Staff analysis of those students that consistently struggle through the course has identified two primary challenging avenues. Students were not adequately prepared procedurally or mentally to begin the study of Precalculus, and/or they chose not to pursue the study and practice required to be successful while in the course.

This gap analysis will focus on the 1st avenue by investigating how to improve the student experience in Trigonometry by focusing on student preparation for the course. Appendix C is a table summarizing the 4 gaps that will be addressed and the initial root cause projections. This avenue will be addressed first as deficiencies resulting from identified gaps are drivers of many of the issues on the 2nd avenue. Successful intervention on the 1st may eliminate, or significantly reduce the gaps found in, the 2nd.

Intervention Determination and Development

Wagner's *Most Likely to Succeed* yields several options for preparing our students for life which correlate well with Trigonometry activities. Combining pursuit of these options with several enhancements to our current teaching strategy should yield the desired positive results. In regard to the information-rich world that our students are entering, Wagner (2015) states, "What matters most in our increasingly innovation-driven economy is not what you know, but what you can do with what you know" (p. 27).

Initial intervention projections documented in Appendix B identify environmental interventions to be the major area of impact with a focus on clearly and concisely communicating the concepts/skills (information) that the students need to acquire as well as educating the students on the benefits of skill acquisition in a quest to drive development of student self-initiative and motivation.

Environmental Intervention

The most detrimental activity to student preparation for Trigonometry is the 'loss' of critical skills over the summer break.

Based on the directive from Wagner in the Intervention preface above, there is a strong case for providing prospective Precalculus students with engaging activities to perform throughout the summer in an effort to keep their previously acquired skills and knowledge activated. This allows them to move quickly to 'do with what you know' when they return to school in September. Environmental intervention is the area of most promise as the information is easily accessible by students, so our intervention actions revolve around clarifying detail and making the material more alluring. Building an engaging summer program is job one.

Making the summer material more enticing to digital natives requires use of a digital platform or venue. An overarching challenge in education is to create a digital citizenship attitude whereby students utilize their digital tools for education as well as entertainment in a comfortable, competent, and responsible manner. Digital tools are a significant cornerstone of their culture moving forward and education needs to breach the application barrier.

In regard to the impact of a summer hiatus from schoolwork, Wagner (2015) shares results of a study conducted at Lawrenceville School, a private college preparatory high school in New Jersey. Upon returning from summer vacation, students took a simplified version of a science final exam that they had taken three months earlier. The average score in June was 87% while the September average dropped to 58%. Though knowledge and skill ‘loss’ during the 3 month hiatus is an expected leading impactor, there are other gaps to investigate as well.

The environmentally-focused interventions to address the gaps are fourfold:

- Design and deploy appealing practices and activities addressing the top 5 challenges for the students to complete throughout the summer.
- Adjust curriculum in pre-requisite courses to better address the top 5 challenges.
- Utilize a digital platform for deployment to influence engagement levels.
- Include parents/guardians in the tracking of summer progress.

The first action to occur will involve the Precalculus instructors identifying the top 5 (labeled the 'high 5') concepts/skills which have historically been the most detrimental pertaining to student preparation for Trigonometry. The 'high 5' are then the focus for the Precalculus instructors to build a summer program in an engaging digital format to be deployed for those students planning to pursue Precalculus in the fall. This will include refresher training on the 'high 5' and intriguing activities to be completed at regular intervals throughout the summer. The more ambitious Precalculus instructors may even go as far as developing a summer gaming league for the Precalculus students to include online competitions that rival other online activities.

A concurrent, and very key, action regarding the 'high 5' is a PreCal instructor collaboration with the Algebra II instructors in the modification of the 2nd semester Algebra II curriculum to ensure that the 'high 5' are addressed effectively. In conjunction with this adjustment, a performance aid (Appendix A) is honed and formative assessments/surveys are administered.

In order to address those students that are not self-motivated and focused on being as prepared as possible for Precalculus in the fall, a program generating active involvement of parents/guardians will need to be enacted. This is best deployed using the relationship channel already in place with the Algebra II instructors.

Emotional Intervention

Wagner (2015) highlights that we need to give our students ‘work worth doing’ because ‘student motivation remains a critical—and largely ignored—issue in education’ (p. 122). This directive encourages educators to build engaging materials to maintain student attention and focus. This is a tall challenge for summer maintenance work.

The focal emotional intervention is building activities in the digital environment that capture the interest of the students and rival their many distractions in that digital space. The initial action is to build lessons and practices based on ‘work worth doing’ and then, ideally, extend to a program that involves a summer-long competition including prizes during the summer, and subsequent pre-course activity grade compensation when they return in the fall. Prizes would range from school tchotchkes to spirit wear to gift cards to free online gaming codes.

Student engagement is the challenge in this intervention. There is an administrative/cultural issue related to student contentedness with ‘just passing’ versus being driven to master concepts and skills while excelling in the course. This is a cultural challenge that will not be addressed as part of this study. It is a campus-level initiative in some form each year at each school in the nation.

Performance Aid Intervention

Though the major gap impacts will be realized in the Environmental and Emotional arenas, a common differentiation technique is to provide performance aids at the point of exposure, yet well in advance of the application. This provides much-needed lead time to those students which require additional time to grasp new concepts/skills.

The opportunity seized is documenting some basic Trigonometry tools, initially introduced in Geometry, which will be revisited during the second semester of Algebra II. During this revisit, a modified version of the performance aid in Appendix: A will be distributed, discussed, and modeled with the students. This will be a tool available to them for assessments regarding basic skill internalization.

Intervention Evaluation

The Kirkpatrick Partners (2016) have provided HPT professionals with a 4-level training evaluation tool which I have applied to the proposed interventions and captured in the table to follow.

Kirkpatrick Level	Performance Aid Intervention	Environmental Intervention	Emotional Intervention
Level 1: Reaction	A formative assessment will be administered immediately following PreCal Tool deployment containing both simple performance questions and survey questions such as: How comfortable do you feel with the	Have Algebra II instructors introduce and demonstrate the activities assigned for the summer. Survey the students following their getting familiar with the program and platform with questions such as: How motivated are you able participating in a	Experiment with rewards systems in the digital space during Algebra II pre-teaching of Precalculus and informally survey and objectively observe the students regarding engagement levels.

	PreCal Tools job aid (1 – 5, where 5 is mastery.) and it's use? Does this introduction to Trigonometry pique your interest in the course?	summer program to help you prepare for a challenging course next fall (1 – 5 where 5 is very motivated)? How likely are you to complete at least 75% of the work in the summer program (1 – 5 where 5 is very likely.)?	
Level 2: Learning	A formative assessment will be administered immediately following PreCal Tool deployment containing both simple performance questions and survey questions such as: Was the PreCal Tools job aid helpful in addressing the problems on this quiz (1 – 5, where 5 is very helpful.)?	Track performance levels throughout the summer by capturing student data from the digital platform and evaluate both progress and accuracy.	N/A
Level 3: Behavior	A formative assessment will be administered near the end of Algebra II containing simple performance questions. Success levels on this assessment will highlight content retention levels.	Issue regular surveys to students and parents/guardians regarding participation and engagement levels to include questions such as: Has the summer program met your expectations (1 – 5 with 5 being exceeded expectations.)? If less than 4, please identify the concern. What enhancements would increase participation and engagement?	Include student surveys as part of summer activities to gather student feedback regarding the personal motivational impact of digital activities and the reward program.
Level 4: Results	Success levels on a formative assessment	Compare results of an assessment administered	Student survey in the fall regarding

	administered near the end of Algebra II containing simple performance questions will reveal the impact of this activity. There may be historical, comparable assessments with which to compare these scores.	in mid-May and a comparable assessment administered during the first week of the fall semester. Comparable or better scores represent success.	their thoughts on the summer activities and rewards. How impactful and enticing were the activities and rewards?

Recommendation Summary

Addressing gaps in student preparation levels for Trigonometry requires activity in two different locations on the 6 month timeline leading to the start of class. The first occurs near the end of their pre-requisite Algebra II course where basic Trigonometry concepts/skills are introduced within the framework of background knowledge. This is accompanied by a performance aid (PreCal Tools) that reinforces the basic tools used in Trigonometry. The second activity occurs throughout the summer where students regularly participate in Trigonometry-based digital activities.

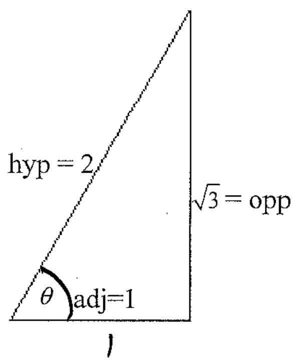
Successful implementation of these two interventions will yield a student open to the challenges of learning Trigonometry, confidently brandishing their PreCal Tools.

Appendix A: Performance Aid

Following is a 4 page job aid to be honed to better tie to background learning and be introduced to students during the 2nd semester of their Algebra II course in order to begin directing their mind towards their 'tools' in Precalculus, the following year. This is currently built as a direct Precalculus support. Modifications should be applied after adjustment of the Algebra II curriculum to provide an information bridge from the Algebra II learning to the application of the Precalculus tools.

S=O/H C=A/H T=O/A

Let $\theta = 60^\circ$

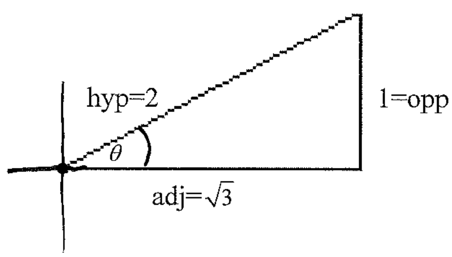


$$\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{1}{2}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Let $\theta = 30^\circ$

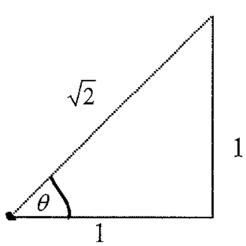


$$\sin\theta = \frac{opp}{hyp} = \frac{1}{2}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{opp}{adj} = \frac{1}{\sqrt{3}}$$

Let $\theta = 45^\circ$



$$\sin\theta = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{opp}{adj} = \frac{1}{1} = 1$$

- For any right triangle, if we were to locate θ at the origin of a Cartesian plane, and let the hypotenuse be the radius (in Quadrant I) of a circle whose center is also at the origin, then the adjacent side becomes the horizontal component, and the opposite side becomes the vertical component.

Therefore, hypotenuse = radius (r) , adjacent side = x, and opposite side = y.

Further,

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ $\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ $\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

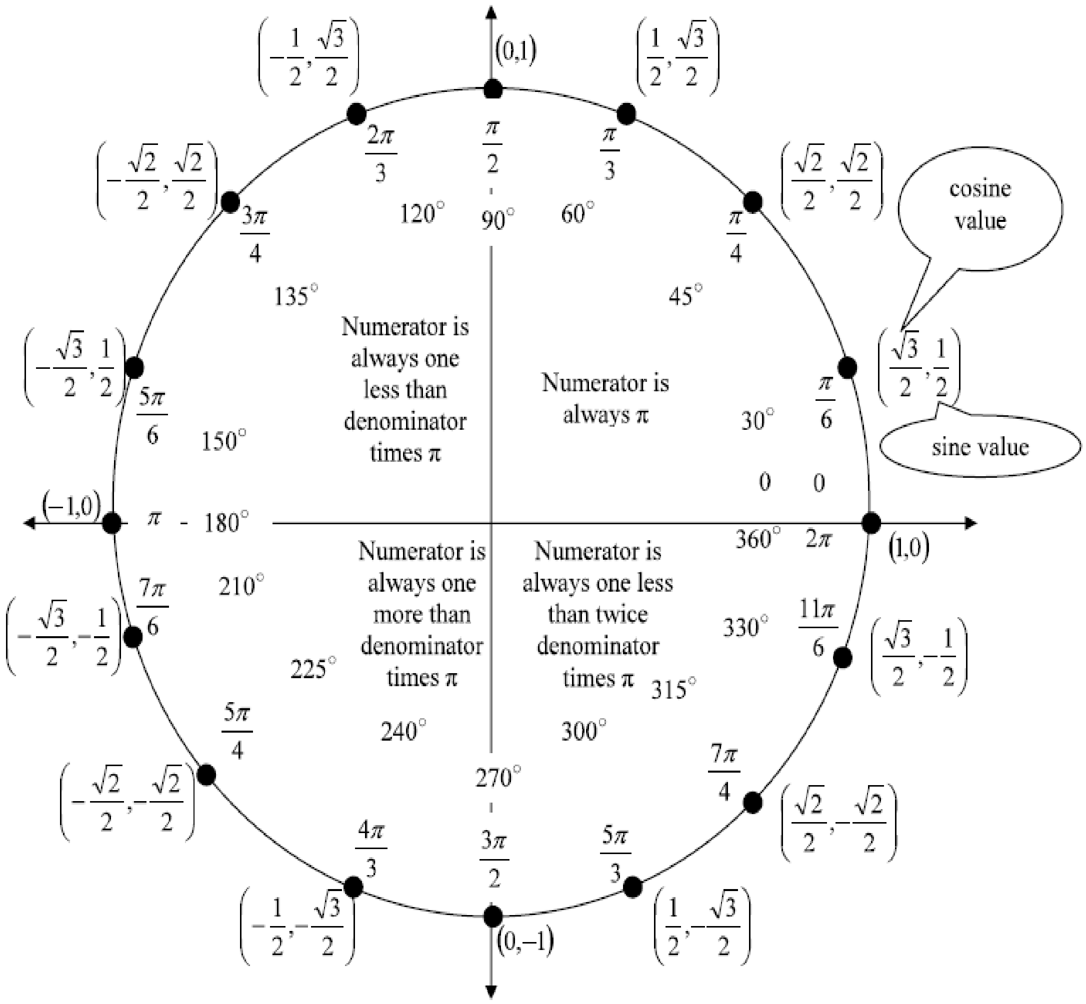
- The reciprocal function of sine θ is cosecant θ and is abbreviated as csc θ
- The reciprocal function of cosine θ is secant θ and is abbreviated as sec θ
- The reciprocal function of tangent θ is ~~cotang~~ cotangent θ and is abbreviated as cot θ

$\therefore$ $\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

- We can reflect the side values for our special right triangles into the other three quadrants (changing their signs as necessary) and have reference triangles in each quadrant. These triangles do not really have negative lengths for sides, as they are for reference only. However, the directed distances of the horizontal and vertical components are represented by positive values or negative values depending on their orientation with respect to the origin.

NOTE: The radius is always positive.

Unit Circle and Reference Angles



Some hints when dealing with radians.

Value of Denominator	Reference Angle
?/6	30°
?/4	45°
?/3	60°

Precalculus Notes Section 5.1– Fundamental Identities, (5.1 Part 1)

Also: $\sin^n\theta = \frac{1}{\csc^n\theta}$ etc.

YOU MUST KNOW THESE IDENTITIES!

The Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

The Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

more → $\tan^n\theta = \frac{\sin^n\theta}{\cos^n\theta}$

$\cot^n\theta = \frac{\cos^n\theta}{\sin^n\theta}$

The Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

The Co-function Identities accomplished by reflecting across y-axis & phase shift right $\frac{\pi}{2}$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

The odd/even Identities Recall symmetry proof • If $f(x) = f(-x)$ → even function & y-axis symmetry

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$ • If $f(-x) = -f(x)$

$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$ then odd function & origin symmetry

These identities are often used to rewrite expressions in terms of predetermined trig function.

For example, let's rewrite the expression $\tan\theta \cdot \cos\theta$ in simplified terms:

$\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1}$

$\sin\theta$

Appendix B: Initial Performance Gap Investigation Projection

Performance Gap: Description	Technique/Tool for further investigation	What you hope to learn
Students 'lose' critical skills over the summer break.	Consider performing an activity akin to the Lawrenceville School benchmarking experiment. Provide students with base knowledge supporting activities to pursue over the summer.	Confirm that Lawrenceville School findings also apply to our student demographic, thereby allowing pursuit of similar solutions. Can we limit knowledge and skill loss by simply providing summer work and communicating with parents regarding support?
Students did not master skills in lower level courses of math.	Student body analysis to determine key concepts that are not mastered.	Identify key concepts that both need to be reinforced via summer work, and where instructional methods in previous courses need to be revisited by those instructors.
Students are not motivated to be successful in math.	Interview students and review published papers to identify how to capture student attention with 'problems worth solving'.	How to rebuild or enhance current instructional methodologies to engage the digital natives and help them to understanding the impact of math on all of our futures.
Though adept at social media, students are not so adept at digital world navigation.	Identify and supply engaging digital educational tools and games to incite (in a good way) students to explore.	What are effective avenues for enhancing educational digital offerings to capture the attention of digital natives? What is an effective method for gaining useful feedback from digital natives regarding current productions?

Appendix C: Initial Performance Gap Analysis

Performance Gap: Description	Desired State	Actual State	Probable root cause(s)	Intervention Category
Students 'lose' critical skills over the summer break.	Students keep current on their math skills over the summer.	Students focus on non-academic activities over the summer.	- Unclear on what to study. - No clear directive to keep current. - Distractions	Environmental Environmental Environmental
Students did not master skills in lower level courses of math.	Students exit their math course each year after mastering the key concepts.	Students 'get' enough knowledge to pass the course.	- Student self-initiative challenges. - Time challenges. - Cascade of misunderstanding.	Environmental Environmental Performance Aid
Students are not motivated to be successful in math.	Students self-initiative drives their acquiring needed knowledge and skills.	Students pursue enough understanding to 'get by' and obtain course credit.	- Clouded vision of value and future applications. - School focus on 'passing'.	Environmental Emotional
Though adept at social media, students are not so adept at digital world navigation.	Students comfortably navigate digitally to acquire knowledge and skills.	Students use digital access for games and social media.	- Digital tools marketed as entertainment. - Social media and games are more engaging than knowledge and skill acquisition.	Environmental Emotional

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